CS 691G Computational Geometry (Fall 2005) **Handout 1**
Homework 1 Due: October 24, 2005

Some problems are easy, some are challenging, some are designed just to make you think about a topic, as they may need extra techniques that we have not yet covered. We just want to see what your geometric intuitions are and how you think about such problems. Do as many of them as you can. When an algorithm is sought, do not attempt the "best" possible one.

1. **Kernel of a star-shaped polygon.** Is the kernel always convex? Either give a counter-example, or sketch a proof that it is so. The proof need not be complete - let your intuition guide you, and we'll work out the details in class later.

2. **Guarding the walls.** Is it true that it suffices to guard the walls of a polygonal gallery? If true, sketch a proof. If not, give an example with guards covering the walls (boundary of the polygon) but not the whole interior.

3. **Deciding if a polygon is star-shaped.** Describe an algorithm for deciding star- shapeness. Analyze your algorithm: is it polynomial time?

4. **Guarding with non-vertex guards.** The original Art Gallery problem asked for the minimum number of guards needed to cover the interior of the polygon. The set of interior points is a continuum. Discretize the problem. More precisely, show that there exists a set of points S so that if there exists a solution with k guards, then there exists a solution with the same number of guards placed at some of the points in S.

5. **Binary trees and polygon triangulations.** Prove or disprove: every rooted binary tree is the dual of a polygon triangulation.

6. **Number of triangulations of a convex polygon.** Work out a few examples and try to formulate a conjecture about the number of triangulations of a convex polygon. The answer is a closed formula you may have seen in a Discrete Math course.

7. **Polygons with the smallest number of triangulations.** Are there polygons with exactly one triangulations? What is the minimum number of triangulations a polygon on n points may have? what is the maximum number, and when is it attained?

8. **Tree rotations and triangulations.** If you are familiar with tree rotations in binary trees, give an interpretation of such a rotation, when applied to the dual of a polygon triangulation.

9. **Interior point test.** Using the orientation test, design a predicate to check when a given point is interior or exterior to a triangle given by its three vertices in ccw order.

10. **Collinearity and Overlap of line segments.** Using the degenerate case of the orien- tation test for triplets of planar points, design a predicate to test when two line segments are collinear, and when they do or do not overlap.

11. **Triangulating a convex polygon.** What is the complexity of triangulating a convex polygon?

12. **Testing for polygon convexity.** Design a linear time algorithm to test if polygon (given by its list of vertices in ccw order) is convex or not.

13. **Monotone polygons.** Can a polygon be monotone with respect to exactly one direction?

14. **Sweeping a polygon with holes.** Sketch an algorithm for triangulating a polygon with holes. The algorithm should triangulate the interior of the polygon, outside the holes. Analyze the time complexity of your algorithm.

15. **3-coloring for triangulations of polygons with holes.** Give an example of a trian- gulation of a polygon with holes which is not 3-colorable. Formulate a conjecture about 3-colorabiity in terms of some combinatorial property of the dual graph of the triangulation.

16. **Partitioning into convex quadrilaterals.** Prove or disprove: every simple polygon with an even number of edges can be partitioned by diagonals into convex quadrilaterals.